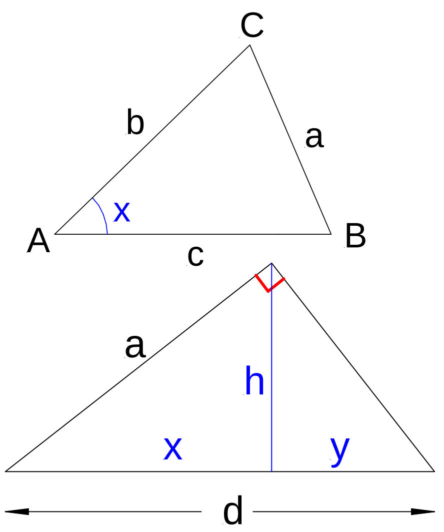
FORMULAIRE DE TRIGONOMÉTRIE

Loi d'Al-Kashi :



Triangle quelconque :

$a^2 = b^2 + c^2 - 2\,b\,c\cos x$

Triangle rectangle :

$x = a^2 / d$

$h^2 = x\,y$

Trigonométrie circulaire réelle :

$\cos x = \cos y \Rightarrow x = \pm y + 2.k.\pi$

$\sin x = \sin y \Rightarrow x = y + 2.k.\pi$ ou $x = (\pi - y) + 2.k.\pi$

$\text{tg}\, x = \text{tg}\, y \Rightarrow x = y + k.\pi$

$\text{cotg}\, x = \text{cotg}\, y \Rightarrow x = y + k.\pi$

$\cos(-x) = \cos x$ | $\sin(-x) = -\sin x$ | $\text{tg}(-x) = -\text{tg}\, x$ | $\text{cotg}(-x) = -\text{cotg}\, x$

$\cos(\pi + x) = -\cos x$ | $\sin(\pi + x) = -\sin x$ | $\text{tg}(\pi + x) = \text{tg}\, x$ | $\text{cotg}(\pi + x) = \text{cotg}\, x$

$\cos(\pi - x) = -\cos x$ | $\sin(\pi - x) = \sin x$ | $\text{tg}(\pi - x) = -\text{tg}\, x$ | $\text{cotg}(\pi - x) = -\text{cotg}\, x$

$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x \qquad \sin\left(\frac{\pi}{2} + x\right) = \cos x \qquad \text{tg}\left(\frac{\pi}{2} + x\right) = -\text{cotg}\, x \qquad \text{cotg}\left(\frac{\pi}{2} + x\right) = -\text{tg}\, x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \text{tg}\left(\frac{\pi}{2} - x\right) = \text{cotg}\, x \qquad \text{cotg}\left(\frac{\pi}{2} - x\right) = \text{tg}\, x$$

$$(\sin x)^2 + (\cos x)^2 = 1 \qquad (\text{tg}\, x)^2 + 1 = \frac{1}{(\cos x)^2} \qquad (\text{cotg}\, x)^2 + 1 = \frac{1}{(\sin x)^2}$$

$$\cos(x + y) = \cos x.\cos y - \sin x.\sin y \qquad \cos(x - y) = \cos x.\cos y + \sin x.\sin y$$

$$\sin(x + y) = \sin x.\cos y + \sin y.\cos x \qquad \sin(x - y) = \sin x.\cos y - \sin y.\cos x$$

$$tg(x + y) = \frac{tg\ x + tg\ y}{1 - tg\ x\, tg\ y} \qquad tg(x - y) = \frac{tg\ x - tg\ y}{1 + tg\ x\, tg\ y}$$

$$\cos(2x) = (\cos x)^2 - (\sin x)^2 = 2.(\cos x)^2 - 1 = 1 - 2.(\sin x)^2$$

$$\sin(2x) = 2.\sin x.\cos x$$

$$tg(2x) = \frac{2.tg\ x}{1-(tg\ x)^2}$$

$$(\cos x)^2 = \frac{\cos(2x)+1}{2} = \frac{1}{1+(tg\ x)^2}$$

$$(\sin x)^2 = \frac{1-\cos(2x)}{2} = \frac{1}{1+(\cot g\ x)^2}$$

$$\sin x.\cos x = \frac{\sin(2x)}{2}$$

$$Si\ on\ pose:\ t = tg\left(\frac{x}{2}\right) \Rightarrow \sin x = \frac{2.t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad tg\ x = \frac{2.t}{1-t^2}$$

$$\cos x + \cos y = 2.\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right) \qquad \cos x - \cos y = -2.\sin\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)$$

$$\sin x + \sin y = 2.\sin\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right) \qquad \sin x - \sin y = 2.\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)$$

$$\sin x.\sin y = \frac{\cos(x-y)-\cos(x+y)}{2}$$

$$\cos x.\cos y = \frac{\cos(x-y)+\cos(x+y)}{2}$$

$$\sin x.\cos y = \frac{\sin(x-y)+\sin(x+y)}{2}$$

$$Arc\sin(x) + Ar\cos(x) = \frac{\pi}{2} \qquad \cos(Arc\sin(x)) = \sqrt{1-x^2}$$

Et pour finir une différentielle : $(\cos x)^3 dx = (1-(\sin x)^2)d(\sin x)$

Trigonométrie circulaire & hyperbolique réelle et complexe :

Par définition : $i^2 = -1$

$$\sin x = \frac{e^{i\,x} - e^{-i\,x}}{2\,i} \qquad sh\;x = \frac{e^{x} - e^{-x}}{2}$$

$$\cos x = \frac{e^{i\,x} + e^{-i\,x}}{2\,i} \qquad ch\;x = \frac{e^{x} + e^{-x}}{2}$$

$$\cos x + i\sin x = e^{i\,x} \qquad ch\;x + sh\;x = e^{x}$$

$$(ch\;x)^2 + (sh\;x)^2 = ch(2\,x) \qquad (ch\;x)^2 - (sh\;x)^2 = 1$$

$$Argsh\;x = \ln(x + \sqrt{x^2+1}\,) \qquad Argch\;x = \ln(x + \sqrt{x^2-1}\,)$$

$$Argth\;x = \frac{1}{2}\ln\frac{1+x}{1-x} \qquad Arg\coth\;x = \frac{1}{2}\ln\frac{x+1}{x-1}$$

$$Soit\;z \in Complexes \qquad z = x + i\,y$$

$$\sin z = \frac{sh(i\,z)}{i} \qquad \cos z = ch(i\,z) \qquad tg\;z = \frac{th(i\,z)}{i}$$

$$e^{z} = e^{x}(\cos y + i\sin y)$$

$$Si\;\;z\;\;est\;défini\;en\;polaire:\;\; z(\rho,\theta) \quad \rho = |z| \quad \theta = Arg(z)$$

$$\ln_C(z) = \ln_R(\rho) + i\,\theta$$

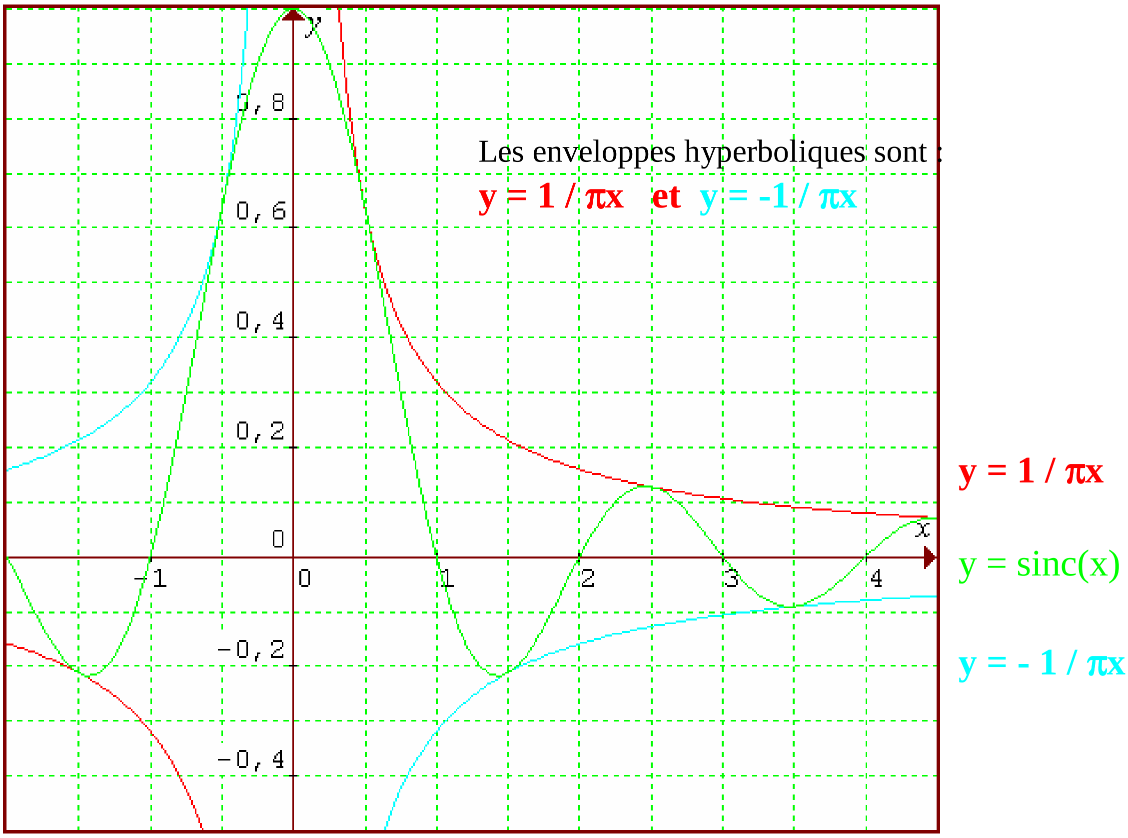
$Quelques\;rappels:$

$$i^{i} = \frac{1}{\sqrt{e^{\pi}}} \qquad \ln(i) = i\,\frac{\pi}{2}$$

$$\forall\,a\;\;\&\;\;\forall\,b \qquad on\;a\;toujours: \quad \log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

$$\Rightarrow \quad \ln(x) = \frac{\log(x)}{\log(e)} \qquad \log(x) = \frac{\ln(x)}{\ln(10)}$$

Sinus cardinal :



sinus cardinal : $sinc(x) = \dfrac{sin(\pi\ x)}{\pi\ x}$ **Pour : $x \neq 0$**

$$\frac{sin(\pi\ x)}{\pi\ x} \xrightarrow[x \to 0]{} 1$$

$si: \quad x \in N \quad \Rightarrow \quad sinc(x) = 0 \qquad sauf \quad pour \quad x = 0$

$$\int_0^{\infty} sinc(x)\ dx \ = \ \frac{\pi}{2}$$

La fonction sinus cardinal admet 2 enveloppes hyperboliques d'équations :

$$y = \frac{1}{\pi\ x} \qquad et \qquad y = \frac{-1}{\pi\ x}$$