Divergents, rotationnels, gradients

$$Laplacien\ de\ U: \quad \Delta U = \frac{(\partial U)^2}{\partial x^2} + \frac{(\partial U)^2}{\partial y^2} + \frac{(\partial U)^2}{\partial z^2}$$

$$\overrightarrow{Grad}\,\vec{U} = \left| \begin{array}{l} \dfrac{\partial U}{\partial x} \\[2em] \dfrac{\partial U}{\partial y} \\[2em] \dfrac{\partial U}{\partial z} \end{array} \right. \qquad \vec{W} = \overrightarrow{Rot}\,\vec{V} \qquad \vec{V} = \left| \begin{array}{l} P \\ Q \\ R \end{array} \right. \quad \Rightarrow \quad \vec{W} = \left| \begin{array}{l} P_1 = \dfrac{\partial R}{\partial y} - \dfrac{\partial Q}{\partial z} \\[2em] Q_1 = \dfrac{\partial P}{\partial x} - \dfrac{\partial R}{\partial x} \\[2em] R_1 = \dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} \end{array} \right.$$

$$div\,\vec{W} = \vec{\nabla}\,\vec{V} \qquad \overrightarrow{Rot}\,\vec{V} = \vec{\nabla} \wedge \vec{V} \qquad \vec{V} = \left| \begin{array}{l} \dfrac{\partial}{\partial x} \\[2em] \dfrac{\partial}{\partial y} \\[2em] \dfrac{\partial}{\partial z} \end{array} \right.$$

$$\overrightarrow{Rot}(\overrightarrow{Grad}\,\vec{V}) = 0 \qquad div(\overrightarrow{Rot}\,\vec{V}) = 0 \qquad div(\overrightarrow{Grad}\,\vec{V}\ ) = \Delta U\ (Laplacien)$$

$$I_S = \iint_S \Omega = \iint_S \left[P_1\, dy \wedge dz + Q_1\, dx \wedge dx + R_1\, dx \wedge dy\right]$$

$$\omega = P\,dx + Q\,dy + R\,dz \qquad d\omega = dP \wedge dx + dQ \wedge dy + dR \wedge dz$$

$Prod.ext.:\ \ \omega_1 = P_1 dx + Q_1 dy + R_1 dz \qquad \omega_2 = P_2 dx + Q_2 dy + R_2 dz$

$$\omega_1 \wedge \omega_2 = -(\omega_2 \wedge \omega_1) \quad \Rightarrow \quad \omega \wedge \omega = 0$$

$$(\omega_1 + \omega_2) \wedge \omega_3 = (\omega_1 \wedge \omega_3) + (\omega_2 \wedge \omega_3)$$

$$(f(x,y,z)\,\omega_1) \wedge \omega_2 = f(x,y,z)\,(\omega_1 \wedge \omega_2)$$

$$\overrightarrow{V_1}\begin{vmatrix} P_1 \\ Q_1 \\ R_1 \end{vmatrix} \wedge \overrightarrow{V_2}\begin{vmatrix} P_2 \\ Q_2 \\ R_2 \end{vmatrix} = \overrightarrow{V_1} \wedge \overrightarrow{V_1}\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} \qquad \begin{matrix} X = (dy \wedge dz)(Q_1 R_2 - Q_2 R_1) \\ Y = (dz \wedge dx)(R_1 P_2 - R_2 P_1) \\ Z = (dx \wedge dy)(P_1 Q_2 - P_2 Q_1) \end{matrix}$$

$$\omega = (dx \wedge dy)(P_1 Q_2 - P_2 Q_1) + (dy \wedge dz)(Q_1 R_2 - Q_2 R_1) + (dz \wedge dx)(R_1 P_2 - R_2 P_1)$$

$$\left(\frac{\partial \overrightarrow{M}}{\partial \alpha} \wedge \frac{\partial \overrightarrow{M}}{\partial \beta}\right) = \left(\frac{\partial \overrightarrow{M}}{\partial u} \wedge \frac{\partial \overrightarrow{M}}{\partial v}\right)\left(\frac{\partial u}{\partial \alpha}\frac{\partial v}{\partial \beta} - \frac{\partial u}{\partial \beta}\frac{\partial v}{\partial \alpha}\right)$$

Formule de Green Rieman :

$\varphi^+ = courbe\ fermée \qquad \Delta = domaine\ intérieur\ à\ \varphi$

$$\int_{\varphi^+} \big(P_1(x,y)\,dx + Q_1(x,y)\,dy\big) = \iint_\Delta \left(\frac{\partial Q_1}{\partial x} - \frac{\partial P_1}{\partial y}\right) dx\,dy$$